

ICMCP 20 March 4, 2024

Quantum analog of Mishchenko-Fomenko
theorem for Ugl.

\mathfrak{g} : Lie 環

\parallel

(e_1, \dots, e_d) 基底

$\mathfrak{g} \subset \mathfrak{g}^*$

$$\overline{\partial \mathfrak{g}} = \{e_1\} \frac{\partial}{\partial e_1} + \dots + \{e_d\} \frac{\partial}{\partial e_d}$$

$\subset \text{hom } \boxed{\mathfrak{S}\mathfrak{g}}$ 对称代数

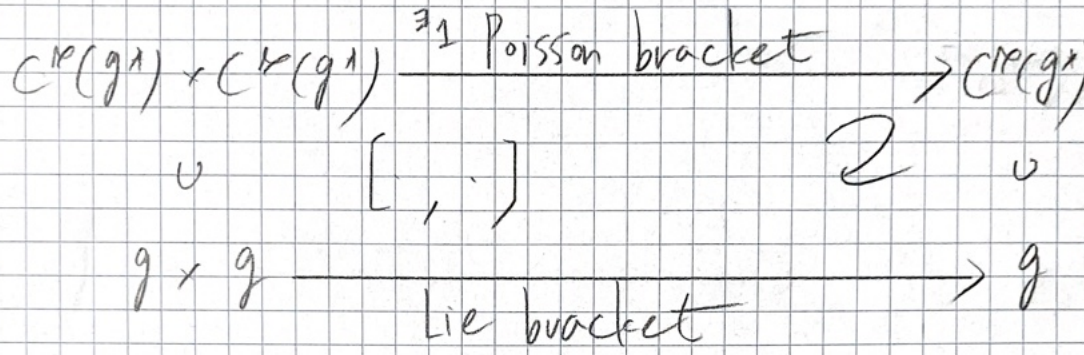
\parallel
 $\mathbb{C}[e_1, \dots, e_d]$

$(\mathfrak{g}^*)^*$

\parallel
 $\mathfrak{g} \subset \mathfrak{S}\mathfrak{g} \subsetneq \mathbb{C}^{\text{No}}(\mathfrak{g}^*)$

linear 多项式代数

Lie bracket + Leibniz



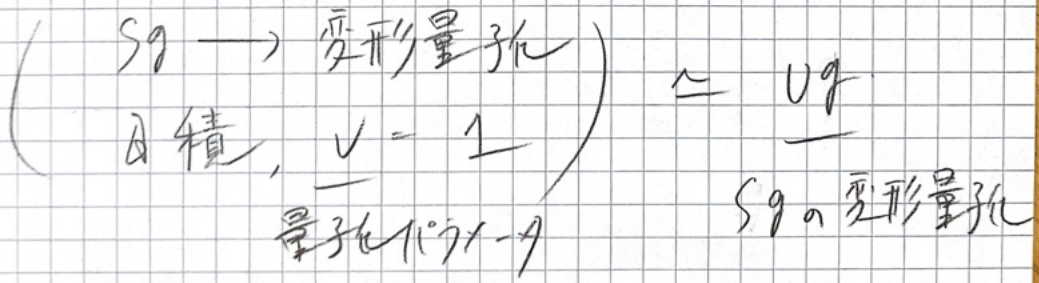
Def 1 \nearrow Lie-Poisson bracket

\mathfrak{g}^* Poisson mfd \rightarrow 変形量子化

Thm (MF, 1998) $\bar{c} \in \mathfrak{S}\mathfrak{g}$: Poisson center

$$\bar{c}_\lambda = \bar{c} [\bar{c}_\lambda x, \bar{c}_\lambda^2 x, \dots : x \in \bar{c}]$$

\bar{c} Poisson 可換



$$g \nu U g = S g$$

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Def 2

C_S : $\{1, \dots, S\}$ 量子 argument shift 代数

$$\Leftrightarrow \begin{cases} C_S \subset U g : \text{可提部分代数} \\ \text{def } g \nu C_S = \overline{C_S} \end{cases}$$

$$S g \quad U g$$

$$\nu \quad \nu$$

$$\overline{C_S} \longrightarrow C_S$$

$$\overline{J_S} \longrightarrow J_S \quad ?$$

$$\mathbb{M} \quad \mathbb{M}$$

$$\text{hom } S g$$

$$\text{hom } U g$$

$$g = g l d$$

$$= g \chi(d, c) = (e_{ij}^{\dot{i}} : i, j \rightarrow 1, \dots, d)$$

$$\mathfrak{gl}_d \cong M(d, \mathbb{C})$$

$$\downarrow$$

$$e_j^i$$

$$\downarrow$$

$$E_j^i$$

$$= \bar{j} \begin{pmatrix} & & & i \\ & & & \\ & & & \\ & & & 1 \end{pmatrix}$$

(j, i) -行列単位

$$e = \begin{pmatrix} e_1^1 & & e_1^d \\ \vdots & & \vdots \\ e_d^1 & & e_d^d \end{pmatrix} \in M(d, \mathfrak{gl}_d)$$

$$\bar{j}_i^i = \frac{\partial}{\partial e_j^i} \in \text{hom } \mathfrak{gl}_d.$$

$$\bar{j} : \mathfrak{gl}_d \longrightarrow M(d, \mathfrak{gl}_d)$$

$$\downarrow$$

$$\downarrow$$

$$x \mapsto \begin{pmatrix} \bar{j}_1^1 x & & \bar{j}_1^d x \\ \vdots & & \vdots \\ \bar{j}_d^1 x & & \bar{j}_d^d x \end{pmatrix}$$

$$1. \quad \bar{j}_i^i \mathbb{C} = \{0\}$$

$$2. \quad \bar{j}_i^i e_j^k = \delta_{ij}^i \delta_j^k.$$

$$3. \quad \bar{j}_i^i(x+y) = (\bar{j}_i^i x) y + x (\bar{j}_i^i y)$$

$Ugl_d = \exists$ 非可换 $\Rightarrow \{r\} \ni r^2 \neq r$ 5
作用素 $\{r\}$.

$$\exists \rightarrow \partial_{\vec{j}}(xy) = (\partial_{\vec{j}}x)y + x(\partial_{\vec{j}}y)$$

$$+ \sum_{k=1}^d (\partial_k^j x)(\partial_j^k y)$$

well-defined (Gurevich-Pyatov-Sapozhouk)

$$\exists \in Ugl_d^* \cong M(d, \mathbb{C}) \quad (\exists_{\vec{j}} = \exists(c_{\vec{j}}))$$

$$\bar{\partial}_{\exists} = \text{tr}(\exists \bar{\partial}) \in \text{hom } Sgl_d$$

$$\partial_{\exists} = \text{tr}(\exists \partial) \in \text{hom } Ugl_d$$

Thm (I-Sharygin) $C \subset Ugl_d = \text{center}$

$$C_{\exists} = C[\partial_{\exists}x, \partial_{\exists}^2x, \dots \quad x \in C]$$

17 \exists 量子 argument shift alg.

(可换, $g \in C_{\exists} = \bar{C}_{\exists}$)

$gl_d^{\mathbb{C}} \subseteq M(d, \mathbb{C})$ 固有值全部是 0 了
 \rightarrow 相似

$$WMA \ \xi = \begin{pmatrix} z_1 & & 0 \\ & \ddots & \\ 0 & & z_d \end{pmatrix} \quad z_i \neq z_j \quad (i \neq j)$$

Vinberg - Rybnikov

$\rightarrow A_{\mathbb{Z}} =$ 可提, $gv A_{\mathbb{Z}} = C_{\mathbb{Z}}$ centraliser

$$\left(\left\{ e_{ii}, \rho_{ii}(\xi) = \sum_{j \neq i} \frac{e_{ij} \bar{e}_{ji}}{z_i - z_j} \right\}_{i=1}^d \right) \textcircled{D}$$

ISTS $C_{\mathbb{Z}} \subset A_{\mathbb{Z}}$ i.e.

$$\forall i, \forall n, \forall x \in \mathbb{C}, [e_{ii}, d_{\mathbb{Z}}^n x] = [\rho_{ii}(\xi), d_{\mathbb{Z}}^n x] = 0$$

$$[d_{\mathbb{Z}}, \text{ad } e_{ii}] = 0$$

$$[e_{ii}, \cdot]$$

direct comp.

Lem

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$$\{x \in U \cap \mathbb{R}^n : \nu(\zeta [\partial q_c(\zeta), \partial x]) = 0\}$$

は C を含む $\partial \zeta$ の不変部分空間

$\partial x, x \in C$ の形 $\rightarrow \exists \eta = \alpha \nu$ となる

Thm 9 証明

$$n > 0, [q_c(\zeta), \partial \zeta^{n-1} x] = 0$$

$$\Rightarrow 0 = \partial \zeta [q_c(\zeta), \partial \zeta^{n-1} x]$$

$$= \underbrace{[\partial \zeta q_c(\zeta), \partial \zeta^{n-1} x]}_C + [q_c(\zeta), \partial \zeta^n x]$$

C

$$+ \nu(\zeta [\partial q_c(\zeta), \partial \zeta^{n-1} x])$$

$$= [q_c(\zeta), \partial \zeta^n x]$$



Lem